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Incorporating catastrophe insurance in power distribution systems investment and planning for resilience enhancement

Qianwen Hu [a,b], Gengfeng Li [b,*], Siyuan Sun [a,b], Zhaohong Bie [a,b]

[a] School of Future Technology, Xi'an Jiaotong University, Xi'an, Shaanxi, 710049, China
[b] State Key Laboratory of Electrical Insulation and Power Equipment, Xi'an Jiaotong University, Xi'an, Shaanxi, 710049, China

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ABSTRACT

Severe damage and high economic losses caused by frequent extreme disasters are becoming unbearable for distribution system operators (DSOs) under climate change. In order to cope with extreme disasters, this paper proposes an investment portfolio and planning framework of multi-resource for resilience enhancement. In the investment portfolio and planning framework, insurance is incorporated to mitigate high economic losses as a risk sharing and resilience enhancement supplementary strategy, and co-optimized with security investment from the perspective of DSOs. Firstly, the insurance investment is modelled based on Gordon-Loeb model according to insurance industry principles and formulated as constraints by piecewise linearization techniques. Secondly, the investment portfolio and planning framework proposed in this paper is modeled as a scenario-based two-stage stochastic mixed integer linear optimization program (MILP). In the first stage, DSO makes investment and planning decisions including energy storage (ESS) allocation, line hardening strategy and insurance investment. Both operation strategies of normal operation scenarios and extreme disaster scenarios under climate change are optimized in the second stage. Finally, progressive hedging (PH) algorithm is applied to solve the two-stage stochastic MILP model. The proposed model and algorithm are tested on the modified IEEE 33-bus test system. The results verify the validity of investment portfolio and planning framework, and the results show that portfolio investment considering insurance can realize resilience enhancement in both load shedding and economic losses.

1. Introduction

1.1. Background and literature review

Climate change is a global problem faced by the development of human society. With the emission of carbon dioxide and the surge of greenhouse gases, the frequency of extreme disasters is also increasing. To this end, governments around the world are strengthening their climate action plans to promote carbon reduction through accelerating the transition to an energy mix dominated by renewable energy sources (RESs) [1]. And more and more distributed RESs gradually connect to the distribution systems (DSs), making the DSs active. But at the same time, it must be noted that frequent extreme disasters have caused huge economic losses to electrical power infrastructure and distribution system operators (DSOs). For example, Hurricane Ian in 2022 caused more than 2 million customers outages in Florida, with more than 99% of customers in several counties without power, causing economic losses as high as $70 billion [2]. Meanwhile, after extreme disasters, power facilities are severely damaged, and DSOs have to spend a lot of money to restore power infrastructure, which usually exceed the actual bearing capacity of DSOs. For example, the power outage repairs in Louisiana caused by Hurricane Laura in 2020 cost up to $1.4 billion [3]. Moreover, it is estimated that tens of billions of dollars need to be spent in post Ian reconstruction work, including repairing and rebuilding power facilities [2]. How to effectively deal with global extreme weather events under climate change has become one of the top ten scientific issues for human social development in 2022 [4].

In order to reduce the severe damage caused by extreme disasters under climate change, an effective solution is to enhance resilience of DSs [5]. Resilience is the ability to prepare for, absorb, adapt to, and rapidly recover from extreme disasters [6]. In the context of global climate change and energy transition, the uncertainty of extreme disasters changes the traditional planning concept of abundance as core. Therefore, in planning stage, the damage to power system caused by low probability and high loss events should be taken into consideration, and

* Corresponding author.
E-mail address: gengfengli@xjtu.edu.cn (G. Li).

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Nomenclature

Indices and Sets

$i,j \in \Omega_N$ Set of node indices i,j
$(i,j) \in \Omega_L$ Set of line indices (i,j)
$\Omega_{SUB} \subset \Omega_N$ Set of nodes connected to substation
$\Omega_{ESS} \subset \Omega_N$ Set of nodes with ESS
$\Omega_{RES} \subset \Omega_N$ Set of nodes with RES
$\Omega_{LT} \subset \Omega_L$ Set of lines with tie switch
$\Omega_{LR} \subset \Omega_L$ Set of lines without tie switch
$t \in T$ Set of time indices t
$s \in S_{EDS}$ Set of extreme disaster scenarios
$s \in S_{NOS}$ Set of normal operation scenarios

Parameters

ψ_{ave} Average occurrence of extreme disasters in a year
C_{ess} Per-unit cost for energy capacity of installing ESS (\$/kWh)
C_{pss} Per-unit cost for power capacity of installing ESS (\$/kW)
C_h Per-unit cost for hardening lines (\$/pole)
P_{r0} Initial premium (\$)
γ Insurance rate
κ Insurance indemnity ratio
$\mathbb{C}$ Total investment budget (\$)
N_L Number of lines allowed to be hardened
N_{ESS} Number of ESSs allowed to be allocated
$p_r(s)$ Probability of typical scenarios after scenarios reduction
U_0 Reference voltage magnitude (p.u.)
$S_{ij}^{\max}$ Maximum power capacity (kVA) of line (i,j)
ω_j Penalty cost (\$/kWh) of unit load shedding
σ_t Unit electricity purchase cost (\$/kWh) from superior substation of DSO at time t
α_l Unit loss penalty coefficient
M Sufficiently large enough positive number
r_{ij}, x_{ij} Resistance (p.u.) and reactance (p.u.) of line (i,j)
$\zeta_{ij,t}^{1,s}, \zeta_{ij,t}^{0,s}$ Parameter indicating the damage status of line (i,j) hardened and not hardened, damaged (1) or functional (0) at time t under scenario s
$U_j^{\min}, U_j^{\max}$ Minimum and maximum voltage magnitude (p.u.) of node j
$P_{l,j,t}^s, Q_{l,j,t}^s$ Active (kW) and reactive power (kVar) demand of node j at time t under scenario s
$P_{res,j,t}^s, Q_{res,j,t}^s$ Active (kW) and reactive (kVar) power output of RES at time t under scenario s
$P_{sub}^{\max}, Q_{sub}^{\max}$ Maximum active (kW) and reactive power (kVar) output of substation
$E_{ess}^{\max}, P_{ess}^{\max}$ Maximum storage capacity (kWh) and power capacity (kW) allowed to be allocated
$\mathrm{soc}_{\min}, \mathrm{soc}_{\max}$ Minimum and maximum state of charge of ESS

Variables

C_{ESS} Total investment cost (\$) for energy capacity of ESS
C_{PSS} Total investment cost (\$) for power capacity of ESS
C_H Total investment cost (\$) of line hardening
C_{PR} Total investment cost (\$) of insurance
$C_{loss}(s)$ Total economic loss (\$) caused by typhoon under scenario s
$C_{ope}(s)$ Total operational cost (\$) under scenario s
$E_{ess,i}$ Storage capacity (kWh) of installed ESS at node i
$P_{ess,i}$ Power capacity (kW) of installed ESS at node i
C_{rep}^s Insurance indemnity (\$) under scenario s
$P_{ij,t}^s, Q_{ij,t}^s$ Active (kW) and reactive (kVar) power flow of line (i,j) at time t under scenario s
$P_{g,j,t}^s, Q_{g,j,t}^s$ Active (kW) and reactive (kVar) power output of node j at time t under scenario s
$P_{sub,j,t}^s, Q_{sub,j,t}^s$ Active (kW) and reactive (kVar) power output of substation at time t under scenario s
$P_{cur,j,t}^s, Q_{cur,j,t}^s$ Active (kW) and reactive (kVar) power abandonment of RES at time t under scenario s
$P_{shed,j,t}^s, Q_{shed,j,t}^s$ Active (kW) and reactive (kVar) power shedding of node j at time t under scenario s
$P_{ess,i,t}^s$ ESS active power (kW) of node i at time t under scenario s
$E_{ess,i,t}^s$ ESS energy state-of-charge (kWh) of node i at time t under scenario s
$U_{i,t}^s$ voltage magnitude (p.u.) of node i at time t under scenario s
x_i^{ess} Binary variable representing whether ESS is allocated at node i (1) or not (0)
x_{ij}^h Binary variable representing whether line (i,j) is hardened (1) or not (0)
x_{pr} Binary variable representing whether DSO decides to insure DS (1) or not (0)
$u_{ij,t}^s$ Binary variable representing whether line (i,j) is faulted (1) or not (0) by typhoon at time t under scenario s
$z_{ij,t}^s$ Binary variable representing whether line (i,j) is closed (1) or not (0) at time t under scenario s
$\vartheta_{mn,t}^s$ Binary variable representing whether node m is the parent node of n (1) or not (0)

it is very necessary to enhance the resilience. Hardening lines, installation of distributed generators (DGs) and energy storage systems (ESSs) become widely used measures. Especially under the background of global carbon emissions and energy transition, ESS, as a flexible resource, will be widely used in power systems. It is expected that the global installed capacity of electrochemical ESSs will reach 1194 GWh in 2030 [7]. It is necessary to fully consider the role of ESSs in resilience planning, and there are many literatures on the use of ESSs to enhance resilience of DSs, including mobile ESSs and stationary ESSs. For example, the planning of mobile ESSs is studied in [8]. Considering extreme disaster scenarios and normal operation scenarios comprehensively, the investment cost and operation cost are minimized by optimizing operation strategy and dynamic microgrid formation strategy. However, the capacity of ESSs is fixed and not optimized in the model. Based on this, a three-stage stochastic optimization model is proposed in [9] to optimize the capacity and location of mobile ESSs. Capacity optimization is realized in the first stage, pre-deployment of mobile ESSs is realized in the second stage, and real-time dispatching of mobile ESSs is realized in the third stage. As for stationary ESSs, a two-step optimal planning framework for ESSs to enhance resilience is proposed in [10], both stationary ESSs under normal scenarios and mobile ESSs under emergency scenarios are considered. Moreover, there are many literatures on resources collaborative planning such as DGs and line hardening to enhance resilience by stochastic optimization model. For example, a scenario-based two-stage stochastic optimization model for line hardening, switch configuration and DG planning is proposed in [11]. For the component damage uncertainty under extreme disasters, the damage states of distribution network components under typhoon weather are obtained based on sampling of component fragility curves. Based on [11], the spatial–temporal correlation among planning decisions and uncertainties in the entire failure-recovery-cost process is further considered in [12], and a distribution network planning model oriented to resilience enhancement is proposed. Based on two-stage stochastic optimization model, reference [13] further proposes a three-stage planning framework for resilience enhancement, where the first two stages are modeled as two-stage stochastic optimization model and the preliminary planning scheme is obtained by solving the mixed integer linear programming (MILP) model. In the third stage, the preliminary planning scheme is improved based on information gap decision theory (IDGT) model. For different risks caused by extreme disasters, the planning strategies under different conditional value at risk (CVaR) are studied in [14], and [14] proposes a risk-based resilient DS planning model, which is also modeled as a two-stage stochastic optimization model based on scenarios. In addition, robust optimization models are also studied for resilience-oriented DSs planning in many literatures. However, the robust models tend to consider the worst scenario and the results tend to be conservative.

For existing investment and planning researches on resilience enhancement, the focus is to realize risk adaptation via security investment (i.e., allocation of distributed resources or line hardening). Specifically, risk adaption means that the consequences caused by extreme disasters (such as load shedding or power outages) can be changed and reduced by security investment. The above papers aim at the minimum blackout time, the fastest recovery time or the minimum load shedding under extreme disasters, and realize effective enhancement of resilience by security investment. The security investment in this paper means the sum of all kinds of resources (i.e., distributed resources, line hardening and so on) for resilience enhancement in which DSOs invest to adapt to the risk of extreme disasters. However, the high economic losses caused by extreme disasters are unbearable for DSOs and ignored in the previous researches. Even though after the reinforcement, DSOs still have to bear high repair costs and economic losses after disasters. How to mitigate high economic losses of DSOs as much as possible while enhancing resilience is an urgent question to be answered. Therefore, catastrophe insurance as a risk sharing strategy in risk management can realize the risk mitigation of high economic losses under extreme disasters [15], there are a few literatures focusing on this field. Insurance is proposed as a risk sharing strategy in [16] to co-optimize with security investment to mitigate infrastructure cybersecurity risk based on Gordon-Loeb model. Insurance investment under different degree losses is discussed in detail. But this paper only analyzes the optimal investment strategy on cybersecurity risk, and does not consider post-disaster insurance indemnity. In [17], for frequent cyber-attack events, the Stackelberg Security Game is used to deploy defense resources, and reference [17] proposes insurance as a risk sharing strategy to transfer cybersecurity risk. The initial premium calculation method is also designed in [17]. In addition, references [15] and [18] also study how to calculate reasonable initial insurance premiums. Reference [15] proposes insurance as a supplementary strategy for resilience enhancement to realize risk management of catastrophe, the initial insurance premium is calculated by a four-module resilience assessment framework. In [18], a catastrophe bond scheme based on insurance is further proposed and optimized to realize cyber risk management of power system.

The insurance mechanisms studied in above literatures to enhance resilience are mainly from the perspective of the insurers, and initial insurance premium is determined by assessing the losses caused by extreme events. Only few studies combine insurance investment with security investment strategies from the perspective of the insured (i.e., DSO). For the insurers, they usually set high initial insurance premiums to maintain profits due to the risk of bankruptcy caused by high compensation under extreme disasters. But high initial insurance premiums are very uneconomical for the insured and result in extremely low insurance willingness [19]. Therefore, many countries around the world support catastrophe insurance by appropriate catastrophe insurance business model [19]. Under different catastrophe insurance business models, governments often decrease the bankruptcy risk of insurance companies and increase the insurance willingness of policyholders through policy incentives or reinsurance. DSOs can mitigate high losses under extreme disasters by reasonable insurance investment as a risk sharing and resilience enhancement supplementary strategy. The analysis of insurance investment from the perspective of the insured (i.e., DSO) can help determine whether DSO should invest in insurance under limited budgets, and can also help governments or insurance companies to make appropriate catastrophe insurance business model and reasonable insurance premiums for the insured to enhance insurance willingness which can realize risk management of extreme disasters [20,21]. So, it is an issue worth discussing how to rationally allocate funds for security investment and insurance investment, and develop an optimal resilience-oriented investment portfolio strategy for DSOs under limited budget to realize not only risk adaption but also risk mitigation.

1.2. Contribution and article organization

Addressing climate changes and natural disasters requires adaptation as well as mitigation, it is necessary to consider the unbearable economic losses of DSOs caused by extreme disasters. Therefore, this paper proposes incorporating insurance into resilience-oriented DS planning as a risk sharing and resilience enhancement supplementary strategy to mitigate high economic losses. Based on previous studies, the main problem we focus on in this paper is how to develop resilience-oriented multi-resources investment and planning strategies to realize not only risk adaptation but also risk mitigation with limited budget to address extreme disaster under climate change. This decision-making process needs considering cost-benefit to realize trade-off between multiple resources and multiple objectives, and develop an optimal investment portfolio strategy which can achieve the full utilization and collaboration of multiple resources. The meaning of "investment portfolio" in this paper is that DSOs should make strategic trade-off among multiple resources including security investment and insurance investment, and develop an optimal portfolio under limited budgets for resilience enhancement. The main contributions of this paper are summarized as follows:

(1) Insurance is incorporated as a risk sharing and resilience enhancement supplementary strategy in the planning stage to mitigate high economic losses caused by extreme disasters. In this paper, insurance investment is modeled based on Gordon-Loeb model according to insurance industry principles. Meanwhile, insurance investment is co-optimized with security investment from the perspective of DSOs, which can enhance resilience in both load shedding and economic losses.
(2) A resilience-oriented investment portfolio and planning framework is proposed in this paper to develop an optimal portfolio strategy that can achieve the trade-off among multiple resources. The proposed framework considers entire normal-failure-recovery-loss process to address climate change and is modeled as a two-stage stochastic MILP model based on scenarios.
(3) To solve the solution difficulty with the increasing of scenarios, progressive hedging (PH) algorithm is applied to solve the proposed two-stage stochastic MILP model based on scenarios. Numerical results show that PH algorithm can adapt to more scenarios and improve computational efficiency by decomposing original problem into several scenario-based sub-problems for solving.

The remainder of this paper is organized as follows. Section II introduces the investment portfolio and planning framework proposed in this paper. Section III models the framework as a two-stage stochastic MILP model. Section IV presents the case study to validate the effectiveness of proposed method. Finally, Section V summarizes the full paper.

2. Investment portfolio and planning framework

2.1. The planning framework and decision variables

The changes brought about by climate change include: (1) a high proportion of RES is connected to grid and distributed RESs make DS active. (2) Climate change leads to frequent extreme disasters. Therefore, the influence of above factors should be considered and the whole process should be modeled when developing the investment and planning strategy for resilience enhancement to cope with climate change. As shown in Fig. 1, DSO makes investment and planning strategy in t_0 to t_1 considering entire normal-failure-recovery-loss process from t_1 to t_4. Specifically, from t_1 to t_2, DS is under normal operation scenarios. This stage is characterized by a high proportion of RESs. ESSs realize the efficient consumption of renewable energy and the minimum operation cost of DSO by optimizing operation strategy. From t_2 to t_3, the occurrence of extreme disasters has caused severe damages to DS. DSO minimizes the losses of power outage by optimizing operation strategy. From t_3 to t_4, high economic losses can be significantly mitigated by insurance indemnity. Therefore, the challenge of climate change can be effectively addressed by taking the factors of whole process from t_1 to t_4 into consideration in the planning stage.

By considering the factors of whole normal-failure-recovery-loss processes, the investment portfolio and planning framework proposed in this paper is modeled as a scenario-based two-stage stochastic optimization model. The structure of two-stage investment portfolio and planning model is shown in Fig. 2. In the first stage, DSO develops investment portfolio and planning strategies. The decision variables are the capacity and location of ESSs, the strategy of line hardening, and the insurance investment strategy. In the second stage, both normal operation scenarios and extreme disaster scenarios are considered. Under extreme disaster scenarios, the economic losses caused by extreme disasters are minimized by optimizing operation strategy and charging-discharging strategy of ESSs. The decision variables include substation output, ESS output, topology reconfiguration and insurance indemnity after disaster. Under normal operation scenarios, the charging and discharging strategy of ESSs is optimized by minimizing the electricity purchase cost and considering the demand of renewable energy consumption. The decision variables include substation output, energy storage output and renewable energy curtailment.

2.2. Uncertainty modeling and scenarios generation

2.2.1. Uncertainty of RES under normal operation scenarios:

For the uncertainty of wind, Weibull distribution is used to approximate the probability distribution of wind speed [22]-[23], which is shown in (1).

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp^{-\left(\frac{v}{c}\right)^{k}} \tag{1}$$

where $f(v)$ represents the probability density function of wind speed; k and c are the shape parameter and scale parameter, respectively; v is wind speed.

The relationship between wind speed and output power is shown in (2).

$$P_{\mathrm{W}} = \begin{cases} 0, v < v_{\mathrm{ci}} \text{ or } v \geqslant v_{\mathrm{co}} \\ \dfrac{v - v_{\mathrm{ci}}}{v_{\mathrm{rate}} - v_{\mathrm{ci}}} P_{\mathrm{rate}}, \quad v_{\mathrm{ci}} \leqslant v < v_{\mathrm{rate}} \\ P_{\mathrm{rate}}, \quad v_{\mathrm{rate}} \leqslant v < v_{\mathrm{co}} \end{cases} \tag{2}$$

where P_{rate} is the rated capacity of wind turbine; v_{ci}, v_{rate} and v_{co} represent the cut-in wind speed, rated wind speed and cut-out wind speed, respectively.

Because Latin Hypercube Sampling (LHS) can realize more efficient sampling and require much smaller sample sizes than other random sampling methods. Meanwhile, the sampling points of LHS can cover each sampling regions and LHS can obtain more reliable results [24]. So, LHS is used to sample wind speed and generate wind speed scenarios. Then k-means clustering algorithm [25] and simultaneous backward reduction (SBR) algorithm [24] are used to reduce scenarios to generate typical scenarios. Firstly, the original scenarios are classified by k-means clustering algorithm. Secondly, SBR algorithm is applied to reduce the scenarios in each class. The scenario reduction technique used in this paper is shown in **Algorithm 1**, where s_i represents scenario vector, p_i represents probability of scenario occurrence.

Algorithm1 Scenarios Reduction Algorithm

Step 1 k-means clustering according to the function of "kmeans" in Matlab:
Set $N = 100$, $K = 5$
Obtain the scenarios n_k in each class k
Step 2 calculating the probability of each typical scenario:
$p_k = n_k/N, k \in K$
Step 3 SBR for obtaining typical scenario in each class k:
For $k = 1$: K
Step 3.1 Find scenario j closet to i:

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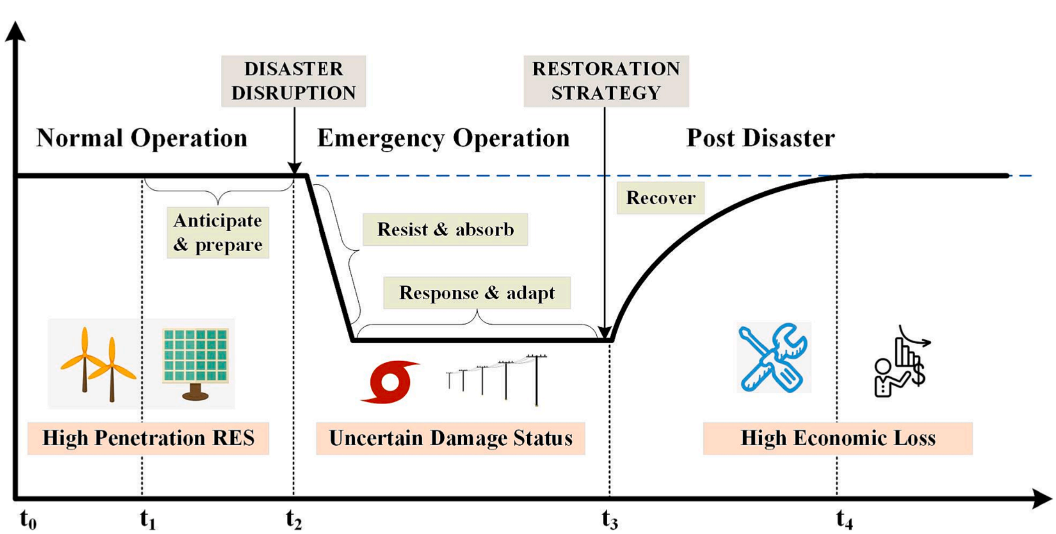

Fig. 1. Whole normal-failure-recovery-loss process to address climate change.

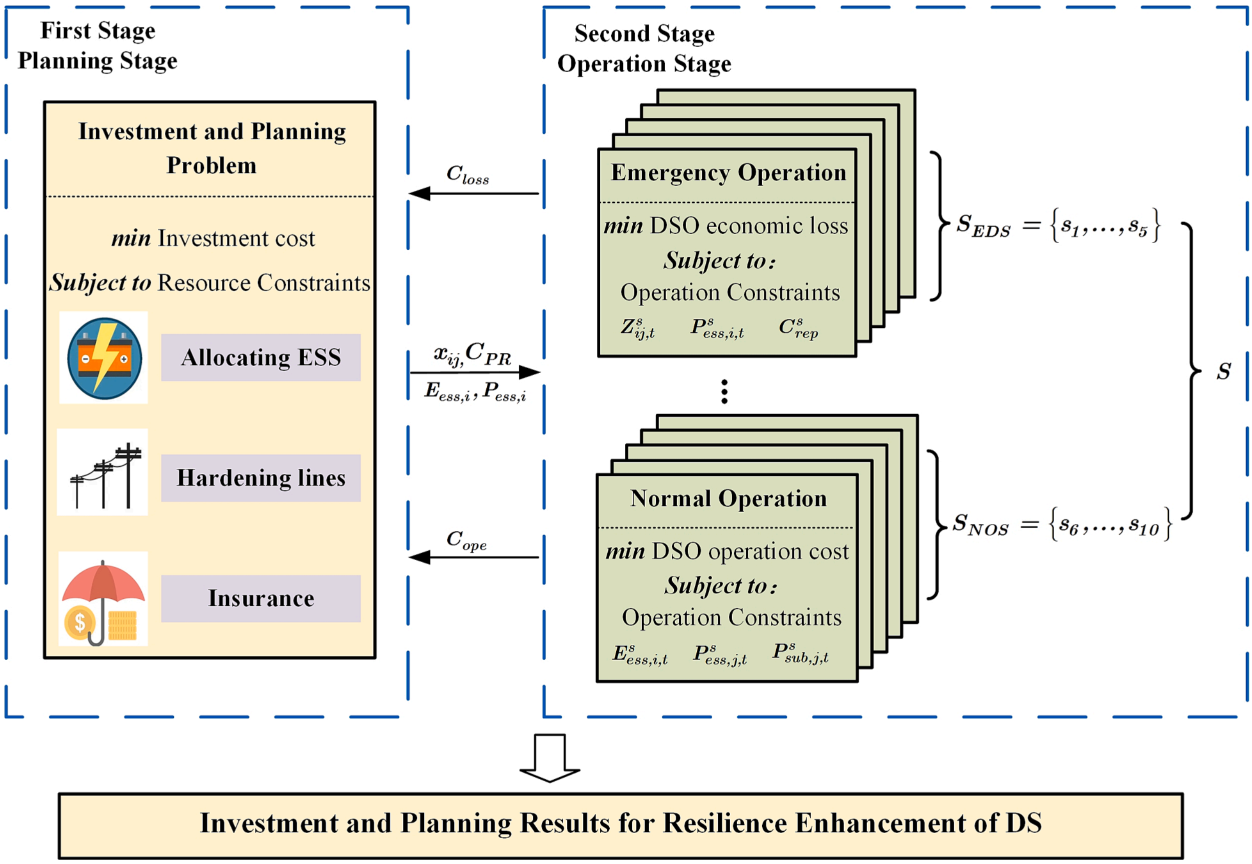

Fig. 2. Two-stage investment portfolio and planning model.

(*continued*)

Algorithm1 Scenarios Reduction Algorithm

$d_{ij} = \min(D_{ij}), \forall i,j \in n_k, j \neq i$
$D_{ij} = \|s_i - s_j\|_2, \forall s_i, s_j \in S_k$
Step 3.2 Find scenario d, satisfies:
$p_d d_{dj} = \min(p_i d_{ij}), \forall d \in n_k$
Step 3.3 Delete scenario d and update scenario probability:
$n_k = n_k - \{d\}$
$p_j = p_j + p_d$
Step 3.4 If $n_k \neq 1$, go to Step 3.1 and continue
Else, next step
End

2.2.2. Uncertainty of line damage status under extreme disaster scenarios

The extreme natural disaster considered in this paper is typhoon. During typhoon disasters, the overhead distribution lines including distribution poles and the conductors between poles are the most vulnerable to be damaged. The magnitude of wind speed will lead to different failure probability, and the wind speed at each component is related to the distance from typhoon eye [26]. Modeling typhoon is not the focus of this paper, we obtain wind speed using the method in [27], where a simulated typhoon generator and Batts typhoon model is used to calculate wind speed. The failure probability of overhead distribution lines under a specific wind speed v is calculated as:

$$P_l[v(t)] = 1 - \prod_{i=1}^{N_l}\{1 - P_p[v(t)]\}\prod_{j=1}^{M_l}\{1 - P_c[v(t)]\} \tag{3}$$

where $P_l[v(t)]$ is the failure probability of overhead line l at time t under wind speed v; $P_p[v(t)]$ is the failure probability of poles and $P_c[v(t)]$ is the failure probability of conductors which can obtained from fragility curves. N_l and M_l are the total number of poles and conductors of line l. The fragility curves used in this paper is according to [15].

For the uncertainty of line damage status, we compare the failure probability under a specific wind speed $v(t)$ with a random variable ε uniformly distributed between 0 and 1. However, it should be noted that the damage status of line is also related to whether the line is hardened or not. When line is hardened, the failure probability of line will be reduced. Thus, two binary variables $\zeta^{1,s}_{ij,t}$ and $\zeta^{0,s}_{ij,t}$ are introduced to represent the damage state of lines hardened and not hardened according to [11]. The state of line under extreme disaster scenario s can be obtained by sampling $\zeta^{1,s}_{ij,t}$ and $\zeta^{0,s}_{ij,t}$, which is shown in (4) and (5). When line is hardened, the probability of failure will reduce to 1/10 of the before value. And if ε is smaller than the probability of failure, the line is damaged.

$$\zeta^{0,s}_{ij,t} = \begin{cases} 0, if\,\varepsilon > P_l[v(t)] \\ 1, if\,\varepsilon < P_l[v(t)] \end{cases} \tag{4}$$

$$\zeta^{1,s}_{ij,t} = \begin{cases} 0, if\,\varepsilon > \dfrac{P_l[v(t)]}{10} \\ 1, if\,\varepsilon < \dfrac{P_l[v(t)]}{10} \end{cases} \tag{5}$$

where ε is a uniform random number between 0 and 1; $\zeta^{0,s}_{ij,t}$ is line damage status which is not hardened, and 1 represents faulty; $\zeta^{1,s}_{ij,t}$ is line damage status which is hardened, and 1 represents faulty.

According to the spatial distribution of load shedding, scenario reduction technique shown in **Algorithm 1** is also used to reduce scenarios to generate typical scenarios.

2.2.3. Insurance investment model

Extreme disasters have the characteristics of “low probability and high loss", the losses often exceed actual bearing capacity of the insured. Catastrophe insurance is an important measure in catastrophe risk management. For the insured, insurance is an investment strategy that can help policyholders transfer and share risk with the insurer and mitigate the losses caused by extreme disasters. For the insurers, they can make use of the time difference between the premium received time and the actual indemnity time to invest premium received, and profit is realized for the insurers through investment income of premiums received [28]. Insurance premiums are the amount of money an individual or business must pay for an insurance policy, which are based on system security risk of policyholders [29]. As shown in Fig. 3, when security investment increases, the security risk of system will decrease which will also result in the decrease of insurance premiums [29].

From the perspective of DSOs, security investment will enhance the resilience of system and reduce the risk caused by extreme disasters, so insurance premiums will be reduced accordingly. However, before solving the planning model, it is not known how much the actual security investment will be. Security investment and insurance premiums are mutually coupled, so this process should be modeled in the planning model considering insurance. In this paper, Gordon-Loeb model is used to model this process according to insurance industry principles. Gordon-Loeb model is a classical mathematical and economic model to analyze the optimal security investment level [30]. Gordon-Loeb model analyzes the security investment, potential loss, vulnerability and system risk from the perspective of economics. The Gordon-Loeb model uses the security breach probability function $S(\xi,\lambda)$ to model the relationship between residual risk and security investment, the security breach probability function should satisfy conditions (6)-(10), and the security breach probability function is shown in (11).

$$S(\xi,0)=0 \tag{6}$$

$$S(0,\lambda)=\lambda \tag{7}$$

$$\frac{\partial S(\xi,\lambda)}{\partial \xi}\leqslant 0 \tag{8}$$

$$\frac{\partial^2 S(\xi,\lambda)}{\partial^2 \xi}\geqslant 0 \tag{9}$$

$$\lim_{\xi\to\infty} S(\xi,\lambda)=0 \tag{10}$$

$$S(\xi,\lambda)=\lambda^{\alpha\xi+1},\alpha>0 \tag{11}$$

where ξ is the total security investment;$S(\xi,\lambda)$ represents system residual risk under a given security investment ξ;λ is a parameter, which represents system vulnerability; α is a parameter, which represents effectiveness of security investment.

As shown in Fig. 3, the relationship between residual risk and security investment is described by security breach function, and then the discounted premium $P_r^{'}$ is obtained under residual risk $S(\xi_1,\lambda)$. For DSOs, a certain proportion of insurance investment can be made to mitigate post-disaster economic losses. The specific insurance investment constraints based on Gordon-Loeb model will be introduced in Section III.

3. Mathematical formulation

3.1. The first stage problem

The objective function (12) of first stage is to minimize the sum of total investment costs, the expected economic losses under extreme disaster scenarios and the expected operation costs under normal operation scenarios.

$$\min C_{inv}+\psi_{ave}\sum_{s\in S_{EDS}}p_r(s)C_{loss}(s)+\sum_{s\in S_{NOS}}p_r(s)C_{ope}(s) \tag{12}$$

$$C_{inv}=C_{ESS}+C_{PSS}+C_H+C_{PR} \tag{13}$$

$$C_{ESS}=\frac{r(1+r)^{T_{EES}}}{(1+r)^{T_{EES}}-1}\alpha_{ess}C_{ess}\sum_{i\in\Omega_{ESS}}x_i^{ess}E_{ess,i} \tag{14}$$

$$C_{PSS}=\frac{r(1+r)^{T_{EES}}}{(1+r)^{T_{EES}}-1}\alpha_{pss}C_{pss}\sum_{i\in\Omega_{ESS}}x_i^{ess}P_{ess,i} \tag{15}$$

$$C_H=\sum_{(i,j)\in\Omega_L}C_h x_{ij}^h \tag{16}$$

$$C_{ESS}+C_{PSS}+C_H+C_{PR}\leqslant\mathbb{C} \tag{17}$$

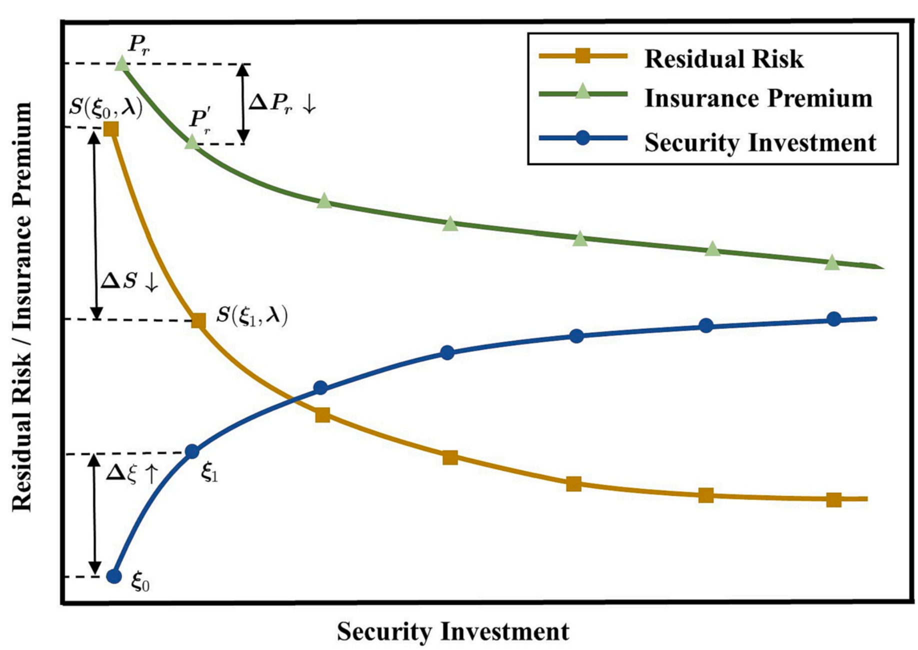

Fig. 3. The relationship between system risk, insurance premium and security investment.

where Eq. (13) is the total investment costs, consisting of allocating ESSs cost (14)-(15), hardening lines cost (16), and insurance investment cost (22). The investment of ESSs is calculated by annualized investment cost, and r is the discount rate; T_{ESS} is the entire investment life of ESSs; Constraints (17) limits the cost of investment cannot exceed budget $\mathbb{C}$.

1) Line hardening investment constraints

$$\sum_{(i,j)\in\Omega_L} x_{ij}^h \leqslant N_L \tag{18}$$

2) ESS investment constraints

$$\sum_{i\in\Omega_{ESS}} x_i^{ess} \leqslant N_{ESS} \tag{19}$$

$$0\leqslant E_{ess,i}\leqslant x_i^{ess}E_{ess}^{\max},\forall i\in\Omega_{ESS} \tag{20}$$

$$0\leqslant P_{ess,i}\leqslant x_i^{ess}P_{ess}^{\max},\forall i\in\Omega_{ESS} \tag{21}$$

3) Insurance investment constraints

$$x_{pr}P_r\leqslant C_{PR}\leqslant x_{pr}P_{r0} \tag{22}$$

$$P_r = P_{r_0}(1-\delta) \tag{23}$$

$$\delta = r(1-S(\xi,\lambda)) \tag{24}$$

$$S(\xi,\lambda) = \lambda^{\alpha\xi+1},\alpha>1 \tag{25}$$

$$\xi = C_{ESS}+C_{PSS}+C_H \tag{26}$$

Constraints (18)-(19) limit the number of lines allowed to be hardened and the number of ESSs allowed to be installed, respectively. Constraints (20) and (21) limit the ESS capacity and power installed at node i, respectively. Constraint (22) is the total insurance investment cost constraint, and x_{pr} represents whether DSO decides to insure DS. If DSO decides to insure, the total insurance investment C_{PR} cannot be less than the discounted premium P_r which is calculated in (23), in order to avoid the insurer to bear excessive financial risk. In addition, the total insurance investment cannot exceed the initial premium P_{r0}, in order to prevent DSO from buying insurance at a high cost to seek insurance company indemnity and reduce security investment. In the first stage, DSO will take a series of measures to enhance the resilience of DS, and system security risk in face of extreme disaster will be reduced, so insurance premiums will be reduced accordingly. The security breach probability function in (25) of Gordon-Loeb model is used to describe the relationship between security investment and system security risk, ξ is security investment including line hardening and ESS allocation cost in this paper. Eq. (24) computes the discount δ that DSO obtains by increasing security investment [16]. δ is based on absolute change, which results in deeper discounts to increase insurance willingness. In (24), the discount rate r determines the final discount, where r is determined by the insurer [16]. For example, if system risk is reduced 30% due to security investment and r is 50%, then the discount δ is finally set as 15%.

3.2. The second stage problem under extreme disaster scenarios

Under extreme disaster scenarios, the charging and discharging strategy of ESS, the operation strategy of DS and the topology reconfiguration strategy are optimized to minimize economic losses caused by extreme disasters. By evaluating the losses caused by disasters, DSO will obtain insurance indemnity from the insurer. In other words, the objective under extreme disaster scenarios including failure-recovery-loss process aims to minimize the economic losses caused by extreme disasters, as shown in (27). Equation (27) calculates the economic losses under extreme disaster scenario s, which equal to the load shedding losses minus insurance indemnity.

$$C_{loss}(s) = \alpha_l\omega_j\sum_{t\in T}\sum_{j\in\Omega_N}P_{shed,j,t}^s - C_{rep}^s,\forall s\in\Omega_{EDS} \tag{27}$$

The constraints under extreme scenarios are as follows, consisting of DS operational constraints, network reconfiguration constraints, ESS operational constraints, line damage status constraints, and insurance indemnity constraints. Since the wind speed of typhoon under extreme disaster scenarios is greater than the cut-out wind speed of wind turbine, RESs are always in off state. So, RES abandonment constraint is not considered in the second stage problem under extreme disaster scenarios.

1) DS operational constraints

The linearized DistFlow model is adopted in this paper according to [31], which is shown in constraints (28)-(31). Eqs. (28)-(29) describe the relationship of power balance at each node. Eqs. (30)-(31) represent the voltage relationship of adjacent nodes through the big M method. Constraints (32)-(33) limit the active and reactive line flow according to the on–off state of line. Constraint (34) limits the node voltage range between upper and lower bounds. Constraints (35)-(36) limit the range of load shedding. Eqs. (37)-(38) describe the net injected active and reactive power at node j, including substation output, distributed RES output and ESS output. Constraints (39)-(40) limit the range of substation output capacity.

$$\sum_{(j,i)\in\Omega_L}P_{ji,t}^s - \sum_{(i,j)\in\Omega_L}P_{ij,t}^s = P_{g,j,t}^s - \left(P_{l,j,t}^s - P_{shed,j,t}^s\right),\forall j\in\Omega_N,t\in T,s\in S_{EDS} \tag{28}$$

$$\sum_{(j,i)\in\Omega_L}Q_{ji,t}^s - \sum_{(i,j)\in\Omega_L}Q_{ij,t}^s = Q_{g,j,t}^s - \left(Q_{l,j,t}^s - Q_{shed,j,t}^s\right),\forall j\in\Omega_N,t\in T,s\in S_{EDS} \tag{29}$$

$$U_{i,t}^s - U_{j,t}^s - \left(r_{ij}P_{ij,t}^s + x_{ij}Q_{ij,t}^s\right)/U_0\leqslant M\left(1-z_{ij,t}^s\right),\forall(i,j)\in\Omega_L,t\in T,s\in S_{EDS} \tag{30}$$

$$U_{i,t}^s - U_{j,t}^s - \left(r_{ij}P_{ij,t}^s + x_{ij}Q_{ij,t}^s\right)/U_0\geqslant -M\left(1-z_{ij,t}^s\right),\forall(i,j)\in\Omega_L,t\in T,s\in S_{EDS} \tag{31}$$

$$-S_{ij}^{\max}z_{ij,t}^s\leqslant P_{ij,t}^s\leqslant S_{ij}^{\max}z_{ij,t}^s,\forall(i,j)\in\Omega_L,t\in T,s\in S_{EDS} \tag{32}$$

$$-S_{ij}^{\max}z_{ij,t}^s\leqslant Q_{ij,t}^s\leqslant S_{ij}^{\max}z_{ij,t}^s,\forall(i,j)\in\Omega_L,t\in T,s\in S_{EDS} \tag{33}$$

$$U_j^{\min}\leqslant U_{j,t}^s\leqslant U_j^{\max},\forall j\in\Omega_N,t\in T,s\in S_{EDS} \tag{34}$$

$$0\leqslant P_{shed,j,t}^s\leqslant P_{l,j,t}^s,\forall j\in\Omega_N,t\in T,s\in S_{EDS} \tag{35}$$

$$0\leqslant Q_{shed,j,t}^s\leqslant Q_{l,j,t}^s,\forall j\in\Omega_N,t\in T,s\in S_{EDS} \tag{36}$$

$$P_{g,j,t}^s = P_{sub,j,t}^s + P_{res,j,t}^s + P_{ess,j,t}^s,\forall j\in\Omega_N,t\in T,s\in S_{EDS} \tag{37}$$

$$Q_{g,j,t}^s = Q_{sub,j,t}^s + Q_{res,j,t}^s + Q_{ess,j,t}^s,\forall j\in\Omega_N,t\in T,s\in S_{EDS} \tag{38}$$

$$0\leqslant P_{sub,j,t}^s\leqslant P_{sub}^{\max},\forall j\in\Omega_{SUB},t\in T,s\in S_{EDS} \tag{39}$$

$$0\leqslant Q_{sub,j,t}^s\leqslant Q_{sub}^{\max},\forall j\in\Omega_{SUB},t\in T,s\in S_{EDS} \tag{40}$$

2) DS network reconfiguration constraints:

In order to maintain the radial structure of DS in the process of topological reconfiguration, $\vartheta^s_{mn,t}$ and $\vartheta^s_{nm,t}$ are two binary variables introduced to restrict the radial topology according to the spanning tree approach in [32–34]. if m is the parent node of n, then $\vartheta^s_{mn,t}$ is 1; if n is the parent node of m, then $\vartheta^s_{nm,t}$ is 1. Eq. (41) limits that the substation nodes have no parent nodes. Constraint (42) limits that other nodes except substation nodes have at most one parent node. Constraint (43) represents that if line ij is closed, m is the parent node of n or n is the parent node of m.

$$\vartheta^s_{mn,t} = 0, \forall (m,n) \in \Omega_{SUB}, t \in T, s \in S_{EDS} \tag{41}$$

$$\sum_{n \in N(m)} \vartheta^s_{mn,t} \leqslant 1, \forall m \in \Omega_N \backslash \Omega_{SUB}, t \in T, s \in S_{EDS} \tag{42}$$

$$\vartheta^s_{mn,t} + \vartheta^s_{nm,t} = z^s_{ij,t}, \forall (i,j) \in \Omega_L, t \in T, s \in S_{EDS} \tag{43}$$

3) ESS operation constraints:

Eqs. (44)-(45) limit the lower and upper energy stored in ESS at node i, which mean that ESS cannot over-discharge or over-charge to ensure the health of ESS. Constraints (46)-(47) ensure ESS charging-discharging power and the stored energy cannot exceed the planning capacity. Eq. (48) describes relationship between the energy stored in ESS and the charging-discharging power of ESS. Eq. (49) limits energy stored in ESS at the initial time to the allowable maximum value, so that ESS can play a greater value and role in coping with extreme disasters.

$$E^{\min}_{ess,i} = \mathrm{soc}_{\min} E_{ess,i}, \forall i \in \Omega_{ESS} \tag{44}$$

$$E^{\max}_{ess,i} = \mathrm{soc}_{\max} E_{ess,i}, \forall i \in \Omega_{ESS} \tag{45}$$

$$-P_{ess,i} \leqslant P^s_{ess,i,t} \leqslant P_{ess,i}, \forall i \in \Omega_{ESS}, t \in T, s \in S_{EDS} \tag{46}$$

$$E^{\min}_{ess,i} \leqslant E^s_{ess,i,t} \leqslant E^{\max}_{ess,i}, \forall i \in \Omega_{ESS}, t \in T, s \in S_{EDS} \tag{47}$$

$$E^s_{ess,i,t} = E^s_{ess,i,t-1} + P^s_{ess,i,t}\Delta t, \forall i \in \Omega_{ESS}, t \in T, s \in S_{EDS} \tag{48}$$

$$E^s_{ess,i,0} = E^{\max}_{ess,i}, \forall i \in \Omega_{ESS}, s \in S_{EDS} \tag{49}$$

4) Line damage status constraints:

Before solving the two-stage stochastic optimization model, it is necessary to generate random fault scenarios under extreme disasters to describe the uncertainty of line damage status. However, the hardening strategy in the first stage will affect the generation of scenarios, because whether the line is hardened will lead to different failure rates, and the hardening strategy cannot be obtained before solving the optimization model. It will be difficult to obtain line damage state under extreme disaster scenarios. Reference [11] effectively addresses this challenge by sampling two random variables. $\zeta^{1,s}_{ij,t}$ and $\zeta^{0,s}_{ij,t}$,which represent line state of hardened and not hardened, respectively. Constraint (50) means that the damage state of line under a specific extreme disaster scenario s can be obtained by sampling $\zeta^{1,s}_{ij,t}$ and $\zeta^{0,s}_{ij,t}$, if line (i,j) is hardened, x^h_{ij} will equal 1 and the state of line (i,j) is determined by $\zeta^{1,s}_{ij,t}$, otherwise, if line (i,j) is not hardened, the state of line (i,j) is determined by $\zeta^{0,s}_{ij,t}$. Constraint (51) limits the damage lines should be open state, and when line (i,j) is not damaged by extreme disasters, it can be closed or open by remotely controlled switches in the process of topology reconfiguration. In this paper, we assume that all lines have remotely controlled switches.

$$u^s_{ij,t} = \left(1 - x^h_{ij}\right)\zeta^{0,s}_{ij,t} + x^h_{ij}\zeta^{1,s}_{ij,t}, \forall (i,j) \in \Omega_L, t \in T, s \in S_{EDS} \tag{50}$$

$$z^s_{ij,t} \leqslant 1 - u^s_{ij,t}, \forall (i,j) \in \Omega_L, t \in T, s \in S_{EDS} \tag{51}$$

5) Insurance indemnity constraints:

Constraint (52) is the insurance indemnity constraint. According to insurance industry principles, insurance indemnity is a comprehensive form of insurance compensation for damages or losses [28], and the insurer compensates the losses of the insured based on insurance premium. According to the calculation method of insurance indemnity in insurance industry, insurance indemnity is usually paid according to the proportion κ agreed in advance, and the total insurance indemnity should not exceed the amount insured. The amount insured is always $1/\gamma$ of insurance premium, and γ is insurance rate.

$$C^s_{rep} = \min\left\{\kappa C_{loss0}, \frac{1}{\gamma} C_{PR}\right\}, \forall s \in S_{EDS} \tag{52}$$

$$C_{loss0}(s) = \alpha_l \omega_j \sum_{t \in T} \sum_{j \in \Omega_N} P^s_{shed,j,t}, \forall s \in S_{EDS} \tag{53}$$

where Eq. (52) represents insurance indemnity under scenario s is always the smaller between κ times the losses caused by extreme disaster and the amount insured, and (53) calculates the losses caused by extreme disaster under scenario s. κ and α_l represent insurance indemnity ratio and unit loss penalty coefficient, respectively.

3.3. The second stage problem under normal operation scenarios

For normal operation scenarios, the cost of purchasing electricity from superior substation is minimized by optimizing the charging and discharging strategy of ESS. At the same time, the demand of renewable energy consumption can also be met by optimizing ESS operation strategy. The objective function (54) is electricity purchase cost under a specific normal operation scenario s.

$$C_{ope}(s) = \sum_{t \in T} \sum_{j \in \Omega_N} \sigma_t P^s_{g,j,t}, \forall s \in \Omega_{NOS} \tag{54}$$

1) DS operational constraints:

Different from the operation constraints of DS under extreme disasters. In normal operation scenarios, without considering random faults of DS, the occurrence of load shedding can be eliminated by optimizing operation strategy of DS. Therefore, the operation constraints are as follows:

$$\text{constraints } (30)-(34),\ (39)-(40) \tag{55}$$

$$\sum_{(j,i) \in \Omega_L} P^s_{ji,t} - \sum_{(i,j) \in \Omega_L} P^s_{ij,t} = P^s_{sub,j,t} + P^s_{res,j,t} - P^s_{cur,j,t} + P^s_{ess,j,t} - P^s_{l,j,t}, \forall j \in \Omega_N, t \in T, s \in S_{NOS} \tag{56}$$

$$\sum_{(j,i) \in \Omega_L} Q^s_{ji,t} - \sum_{(i,j) \in \Omega_L} Q^s_{ij,t} = Q^s_{sub,j,t} + Q^s_{res,j,t} - Q^s_{cur,j,t} + Q^s_{ess,j,t} - Q^s_{l,j,t}, \forall j \in \Omega_N, t \in T, s \in S_{NOS} \tag{57}$$

2) Distribution system topology constraints:

The main function of tie switch in DS is to realize load transfer when line fault occurs. Under normal operation conditions, the tie switch of DS remains open status. Therefore, for normal operation scenarios, the topological reconfiguration of DS through tie switch is not considered.

The line status can be limited by constraint (58)-(59). Constraint (58) indicates that lines with tie switch under normal operation scenarios keep open status. Constraint (59) indicates that lines except tie switch lines under normal operation scenarios keep closed status.

$$z_{ij,t}^{s} = 0, \forall (i,j) \in \Omega_{LT}, t \in T, s \in S_{NOS} \tag{58}$$

$$z_{ij,t}^{s} = 1, \forall (i,j) \in \Omega_{LR}, t \in T, s \in S_{NOS} \tag{59}$$

3) ESS operation constraints:

$$\text{constraints } (45) - (49), \tag{60}$$

$$E_{ess,i,0}^{s} = E_{ess,i,T}^{s}, \forall i \in \Omega_{ESS}, s \in S_{NOS} \tag{61}$$

Constraint (61) represents the initial moment energy stored in ESS should equal to the final moment, this constraint is ignored under extreme scenarios because ESS mainly plays a role of emergency power support [8], in this case, ESS needs to play the maximum role and potential to restore load power supply, thus extreme scenarios allow ESS to completely discharge, if necessary.

4) RES abandonment constraints:

In order to promote the effectively consumption of renewable energy, the power abandonment rate of RESs can be added to the constraints. The μ represents the power abandonment rate of RESs, and constraints (62)-(63) limit that the power abandonment of RESs cannot exceed the allowable level.

$$0 \leqslant \sum_{j \in \Omega_{RES}} \sum_{t \in T} P_{cur,j,t}^{s} \leqslant \mu \sum_{j \in \Omega_{RES}} \sum_{t \in T} P_{res,j,t}^{s}, \forall s \in S_{NOS} \tag{62}$$

$$0 \leqslant \sum_{j \in \Omega_{RES}} \sum_{t \in T} Q_{cur,j,t}^{s} \leqslant \mu \sum_{j \in \Omega_{RES}} \sum_{t \in T} Q_{res,j,t}^{s}, \forall s \in S_{NOS} \tag{63}$$

3.4. Solution algorithm

A mixed integer nonlinear programming problem is formed by objective function (12)-(16) and constraints (17)-(63). It should be noted that nonlinear constraints will be difficult to solve, so some constraints need to be linearized. For constraints (25), the piecewise linearization technique is used for linearization which can be found in [35]. A linear function of n segments in (64) will be formed after piecewise linearization, linearization of (64) can be realized by (65), where δ are binary variables and ω are continuous variables. For constraint (52) which includes min function, two binary variables are introduced to achieve linearization through the big M method, as shown in (66). The linearization method of multiplying binary variable with continuous variable such as constraint (14)-(16) and (22) can be found in [36], which is shown in (67). Where δ represents binary variables and φ represents continuous variables, respectively.

$$\begin{cases} x = \sum_{k=1}^{n+1} \omega_k \eta_k \\ f(x) = \sum_{k=1}^{n+1} \omega_k f(\eta_k) \end{cases} \tag{64}$$

$$\begin{cases} \omega_1 \leqslant \delta_1, \omega_2 \leqslant \delta_1 + \delta_2, \ldots, \omega_n \leqslant \delta_{n-1} + \delta_n, \omega_{n+1} \leqslant \delta_n \\ \omega_1 + \omega_2 + \ldots + \omega_n + \omega_{n+1} = 1 \\ \delta_1 + \delta_2 + \ldots + \delta_n = 1 \end{cases} \tag{65}$$

$$\begin{cases} C_{rep}^{s} \leqslant \kappa C_{loss0}, C_{rep}^{s} \leqslant \frac{1}{\gamma} C_{PR} \\ \kappa C_{loss0} \leqslant C_{rep}^{s} - \mathrm{M}(1 - \chi) \\ \frac{1}{\gamma} C_{PR} \leqslant C_{rep}^{s} - \mathrm{M}(1 - \mu) \\ \chi + \mu \geqslant 1 \\ \chi, \mu \in \{0, 1\} \end{cases} \tag{66}$$

$$\begin{cases} \beta = \delta \cdot \varphi \\ \beta - M(1-\delta) \leqslant \varphi \leqslant \beta + M(1-\delta) \\ \beta - M \cdot \delta \leqslant 0 \leqslant \beta + M \cdot \delta \end{cases} \tag{67}$$

The final model after linearization is a MILP problem, which can be expressed in a compact form as follows:

$$\min_{x,y} \left\{ c^{\mathrm{T}} x + \sum_{s \in S_{EDS}} p(s) \left(f_s^{\mathrm{T}}, y_{sd} \right) + \sum_{s \in S_{NOS}} p(s) \left(g_s^{\mathrm{T}}, y_{so} \right) \right\} \tag{68}$$

$$\text{s.t.} (x, y_s) \in \Theta \tag{69}$$

where x represents investment and planning decision variables, which are scenario independent. $y_s = y_{sd} \cup y_{so}$ represents the second-stage operation decision variables based on scenarios.

The PH algorithm [37] decomposes the original problem (68)-(69) into a subproblem based on scenarios, as shown in Eq. (70)-(71), where $x(1) = x(2) = \ldots = x(S)$ represents the non-anticipativity constraint to ensure that the decisions do not depend on stochastic parameters. In the scenarios-based subproblem, investment and planning decisions are common to all scenarios after continuous iteration by setting reasonable multiplier and penalty coefficient. By decomposition computation of multiple scenarios, the difficulty of solution can be reduced. **Algorithm 2** shows the specific solving processes and steps of pH algorithm.

$$\min_{x,y} \left\{ \sum_{s \in S} c^{\mathrm{T}} x(s) + \sum_{s \in S_{EDS}} p(s) \left(f_s^{\mathrm{T}}, y_{sd} \right) + \sum_{s \in S_{NOS}} p(s) \left(g_s^{\mathrm{T}}, y_{so} \right) \right\} \tag{70}$$

$$\text{s.t.} \begin{cases} (x_s, y_s) \in \Theta \\ x(1) = x(2) = \ldots = x(S) \end{cases} \tag{71}$$

Algorithm2 Progressive Hedging (PH) Algorithm

Step 1 Initialization:
Set $k = 0, w^k(s) = 0$
for $s = 1 : S$
$x^{k=0}(s) = \underset{x(s)}{\operatorname{argmin}} \{ c^{\mathrm{T}} x(s) + f_s^{\mathrm{T}} y_{sd} + g_s^{\mathrm{T}} y_{so} \}$,
end
$\overline{x} = \sum_{s \in S} p(s) x^{k=0}(s)$
Step 2 Iteration update:
$k = k + 1$
Step 3 Update multiplier and decomposition:
for $s = 1 : S$ do
$$w^k(s) = w^{k-1}(s) + \rho \cdot (x^{k-1}(s) - \overline{x})$$
$$x^k(s) = \underset{x(s)}{\operatorname{argmin}} \left\{ c^{\mathrm{T}} x(s) + f_s^{\mathrm{T}} y_{sd} + g_s^{\mathrm{T}} y_{so} + w^k(s) \cdot x(s) + \frac{\rho}{2} \| x(s) - \overline{x} \|^2 \right\}$$
end
$$\overline{x} = \sum_{s \in S} p(s) x^k(s)$$
$$\varepsilon^k = \sum_{s \in S} p(s) \cdot \| x^k(s) - \overline{x} \|^2$$
Step 4 Termination:
If $\varepsilon^k <$ gap: Stop and return $x^k(s)$
Else go to Step 2 and continue

4. Case study

The proposed two-stage investment portfolio and planning model is verified on the modified IEEE 33-bus distribution system. The simulations are performed by Yalmip and Matlab 2020b on a PC with Intel Octa Core (3.0 GHz) and 16 GB RAM. The MILP problems are solved by Gurobi 9.5.2 under an academic license with the default settings.

4.1. Test system and parameter information

As shown in Fig. 4, there is one substation connected to bus 1 with capacity of 10 MVA and two distributed RESs located at node 3, node 33. The capacity of distributed RESs are 10 kW and 20 kW, respectively. We assume that all lines are equipped with remotely controlled switches and there are two tie switch lines (12,22) and (25,29). The reference voltage is 12.66 kV and allowable voltage of each node is 0.9 ~ 1.1p.u. of the reference voltage. The total system load is 3715 kW and 2300 kVar. The more details about IEEE 33-bus distribution system can be found in [38].

4.2. Investment and planning results analysis

100 scenarios are randomly generated and then reduced into 5 typical scenarios by scenario reduction techniques which are shown in **Algorithm 1**. The node load shedding (p.u.) due to line damage under typical extreme disasters scenarios and distributed RESs output power (p.u.) under typical normal operation scenarios are shown in Fig. 5 (a) and (b).

In order to illustrate the necessity of incorporating insurance in DS planning, two cases are studied for comparison. The base case does not consider insurance investment, and the insurance case takes the framework proposed in this paper to optimize investment and planning strategy for resilience enhancement. The results of investment and planning are shown in Table 2. We can find that if only installation of ESS and hardening lines are carried out without considering insurance, the losses of DSO will be doubled. By the framework proposed in this paper, DSO will install ESS of 631 kWh at node 17 and harden lines (11,12), (30,31). The insurance investment of DSO is 356800$, and the losses are only the half of base case.

The SOC curves of ESS under extreme disaster scenarios and normal operation scenarios are shown in Fig. 6 (a) and (b). The main role of ESS under extreme disaster scenarios is providing emergency power support. ESS keeps stored energy at allowed maximum value before the occurrence of extreme disaster. After the occurrence of extreme disaster, power will be supplied to customers by optimizing discharge strategy. Taking Scenario 1 as an example, the charging-discharging strategy and SOC curves of ESS under Scenario 1 are shown in Fig. 7 (a). Line fault happened at 10:00 due to extreme disasters, and ESS restored power supply for load by continuous discharging. At 19:00, ESS has fully discharged the amount of stored energy.

For normal operation scenarios, ESS realizes efficient consumption of renewable energy and minimizes operating costs of DSO by optimizing charging-discharging strategies. The SOC curves under normal operation scenarios and the charging-discharging strategy under scenario 6 are shown in Fig. 6 (b) and Fig. 7 (b).

In order to verify the effectiveness of the proposed investment portfolio and planning framework, two configuration cases are designed for comparison.

- Case 1: in this case, none of nodes are installed with ESS and none of lines are hardened. Only insurance investment is considered. DSO can only use tie switches of DS for topology reconfiguration to realize failover.
- Case 2: in this case, only installing ESS is considered. Hardening lines and insurance investment are not considered. DSO can take topology reconfiguration and optimize ESS operation strategy to realize failover.
- Case 3: in this case, the framework proposed in this paper is adopted for investment and planning, including installing ESS, hardening lines and insurance investment.

The investment and planning results are shown in Fig. 8. From the comparison of case 1 and case 2, it can be found that insurance can significantly mitigate high economic losses of DSO caused by extreme disasters. Compared with case 1 which only considers insurance investment, the total load shedding of other cases reduces, indicating that the installation of ESS and hardening lines can reduce load shedding caused by extreme disasters. Compared with other cases, the portfolio investment and planning strategy proposed in this paper can realize not only the lowest total load shedding but also the lowest economic losses caused by extreme disaster. That is, through the framework proposed in this paper, DSOs can obtain an optimal resilience-oriented multi-resources investment and planning strategies to realize not only risk adaption but also risk mitigation.

The different degree of losses and the proportion of insurance indemnity will affect investment decision of DSOs in insurance. The unit loss penalty coefficient α_l is set from 0.5 to 3.0, and the insurance in-

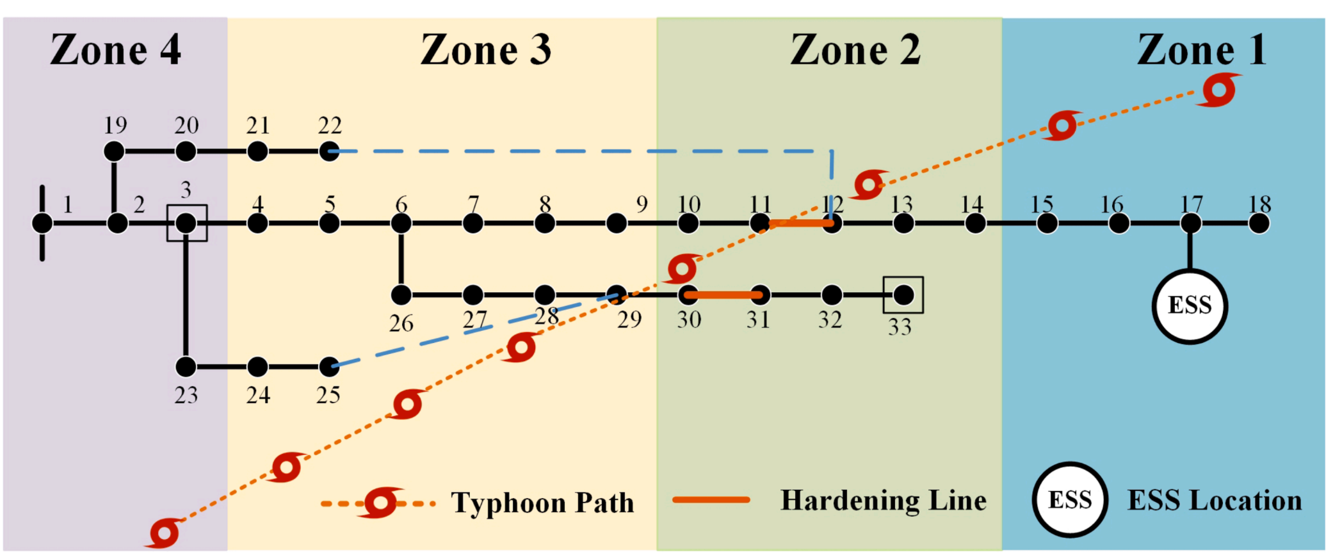

Fig. 4. IEEE 33-bus test system.

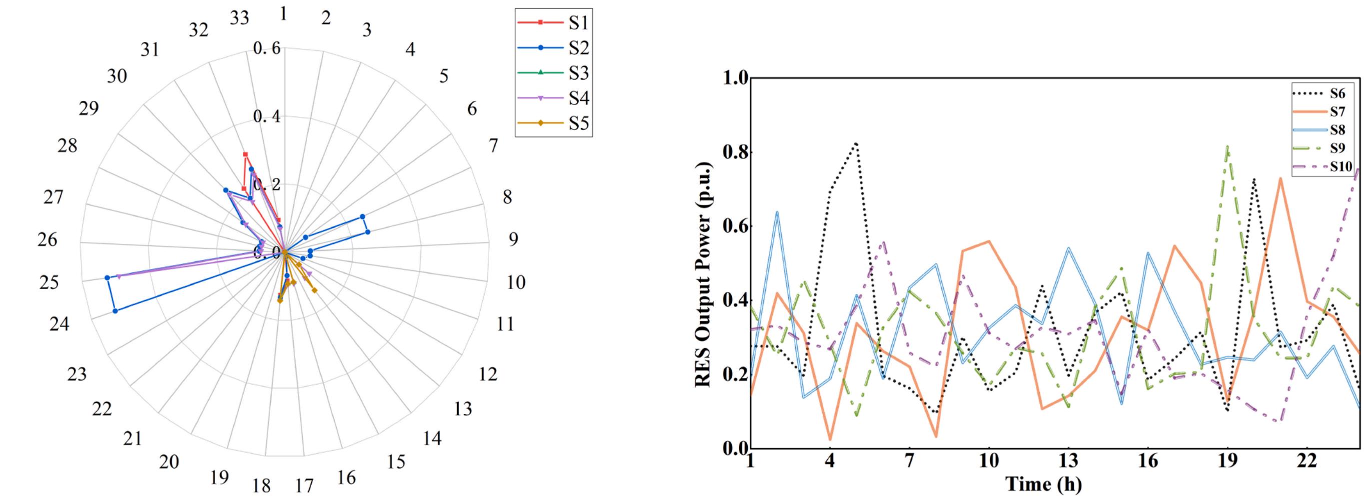

Fig. 5. Typical extreme disaster scenarios (a) and normal operation scenarios (b) after reduction.

Table 1
The capital cost and budget of different measures.

	Cost	Budget
Hardening lines	6000 $/pole	3 lines
Allocating ESS	364 $/kWh	2 ESSs
	145 $/kW	
Initial insurance premium	700,000 $	/
Load shedding penalty	30 $/kWh	/
Electricity purchase cost	0.2 $/kWh	/

The unit investment cost, unit load shedding penalty cost and unit electricity purchase from substation cost are shown in Table 1 according to [8,11], and [39]. The budget in Table 1 means that the number of lines allowed to be hardened is 3 (each line has 2 poles), and the number of ESS allowed to be installed is 2. The discount rate r in this paper is 10% and expected life time is 10 years. The maximum capacity and power of ESSs for installation are 5 MWh and 1 MW, respectively. We assume that annual frequency of extreme disasters ψ_{ave} is 3. $soc_{\min}$ and $soc_{\max}$ are set as 0.1 and 0.9, respectively.

Table 2
Investment and planning results for two cases.

Scheme	Base case	Insurance case
Investment cost	63,088 $	419,878 $
DSO loss	779,370 $	380,000 $
Hardening lines	(11,12), (30,31)	(11,12), (30,31)
Allocating ESS	18:637 kWh 85 kW	17:631 kWh 101 kW
Insurance investment	/	356,800

demnity ratio κ is set from 0.3 to 1, respectively, which is shown in (52)-(53). The total insurance investment results under different unit loss penalty coefficient and insurance indemnity ratio are shown in Fig. 9. It can be found that when the insurance indemnity ratio and the economic losses are low, DSO will not choose to invest in insurance to mitigate economic losses as a resilience enhancement supplementary strategy. This is because when the load losses are too slow, the annual total insurance indemnity obtained by DSO or annual total economic loss caused by extreme disasters is always far less than annual insurance premium, which is very uneconomical for DSO. At the same time, it can be found from the Fig. 8 that when the loss increases, the insurance investment increases sharply. Even the indemnity ratio is low, DSO will still choose investment in insurance to mitigate high economic loss as a resilience enhancement supplementary strategy. This is because extreme disasters with low probability and high loss have caused serious damage to DS, and the total economic losses are difficult for DSO to bear. So, DSO chooses insurance investment as a resilience enhancement supplementary strategy to realize risk mitigation through insurance indemnity. Therefore, as a measure of risk management, insurance is of great significance in the face of extreme disasters with low probability and high loss. However, for disasters with low loss, insurance is often uneconomical compared with security investment.

Due to the limitation of existing technology, ESS investment cost is still high, and it will significantly affect the planning results of ESS. With the breakthrough and progress of technology in the future, the investment cost of ESS will further decrease. In order to explore the impact of ESS investment cost on the results of planning, coefficient α is introduced, which is shown in (14)-(15). The cost coefficient of ESS α_{ess} and α_{pss} are set as 0.6, 1.0 and 1.5 in this paper, and the results are shown in Table 3. It can be found that with the decreasing of ESS investment cost, DSO will install more capacity of ESS for resilience enhancement to cope with climate change. When the investment cost of ESS decreases, the planning power of ESS does not change much compared with the capacity, which indicates that DSO is more inclined to choose ESS with longer energy storage duration. However, when ESS investment cost increases, both capacity and power of ESS decrease significantly. This is because the investment cost of ESS is too high, the investment of ESS for resilience enhancement is uneconomical for DSO.

4.3. Computational efficiency

In order to verify the computational efficiency and performance of PH algorithm proposed in this paper, two solving methods including PH algorithm and direct solution by Gurobi are used for comparison under different number of scenarios, and the results are shown in Table 4. The convergence tolerance gap of PH algorithm is set as 0.1%. The size of ρ will affect the convergence and speed, the penalty coefficient ρ is set according to [34].

For the investment and planning model proposed in this paper, we firstly consider ten typical scenarios, including five normal operation scenarios and five extreme disaster scenarios. We can find that the speed of direct solution by Gurobi is faster than PH algorithm under ten scenarios. However, with the number of scenarios increasing to 20, the solution time of direct solution by Gurobi increases significantly, and the computational efficiency of pH algorithm is better than direct solution by Gurobi. When the number of scenarios is larger, it is impossible to get the running results in a reasonable time directly using Gurobi. Consequently, with the increasing of scenarios, decomposition of the main problem for scenario-based sub-problems through PH algorithm can effectively reduce the computational burden comparing with direct solution, and PH algorithm can also accommodate the feasibility of the

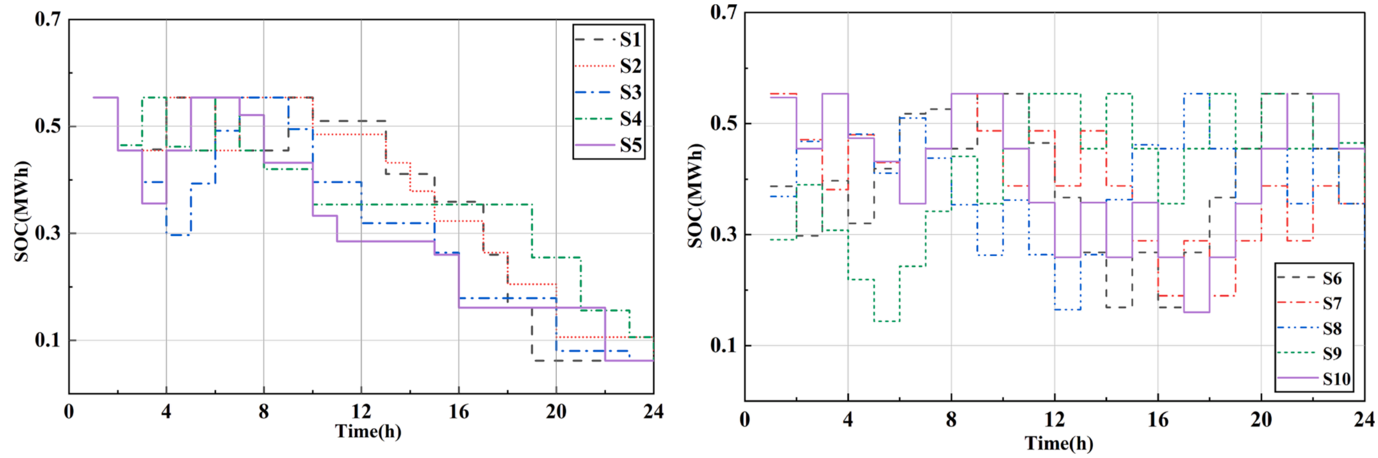

Fig. 6. SOC curves of ESS under extreme disaster scenarios (a) and normal operation scenarios (b).

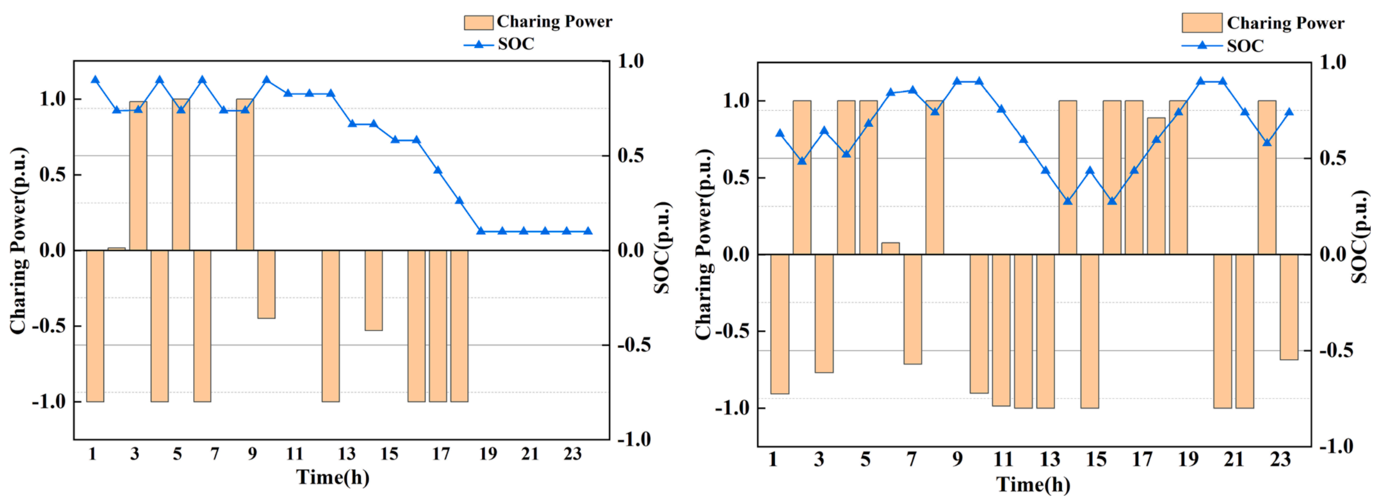

Fig. 7. SOC curves and charging-discharging strategy of ESS under extreme disaster scenario 1 (a) and normal operation scenario 6 (b).

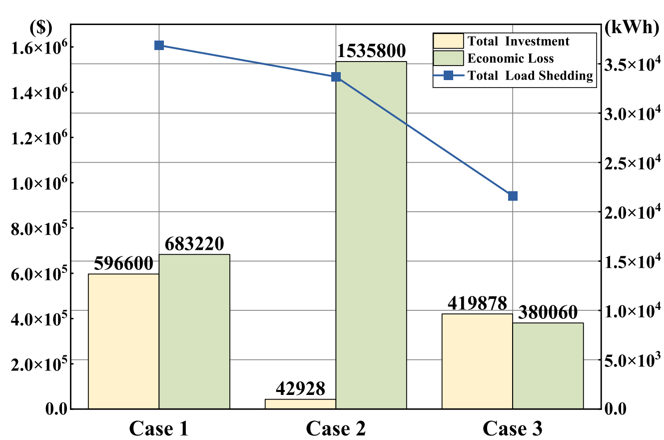

Fig. 8. Investment and planning results under different cases.

model proposed in this paper under more scenarios.

5. Conclusion

This paper proposes a resilience-oriented investment portfolio and planning framework to cope with extreme disasters under climate change, which is modeled as a two-stage scenario-based stochastic MILP model. To reduce computational burden, PH algorithm is applied to solve MILP model. The numerical experiments are performed on the modified IEEE33-bus system. The results verify the effectiveness of the proposed framework and algorithm: (1) For the risk of high economic losses under extreme disasters, incorporating insurance as a risk sharing strategy and resilience enhancement supplementary strategy in the planning stage can help DSOs significantly mitigate high economic losses; (2) The investment portfolio and collaborative planning considering ESS allocation, line hardening and insurance can obtain an optimal resilience enhancement in both load shedding and economic losses. That is, investment portfolio and collaborative planning considering multiple resources including insurance can realize not only risk adaption but also risk mitigation to enhance resilience under extreme disasters; (3) The results also verify PH algorithm can improve computational efficiency under more scenarios. Moreover, our paper can also provide reference for governments and the insurers to better promote the development of catastrophe insurance by analyzing the optimal security investment and insurance investment from the perspective of DSOs. Further research is also needed in the future, e.g., (1) we need more historical data of disasters to support more rational investment decisions; (2) the progress of diversified energy storage (hydrogen storage, heat/cold storage) and long-term energy storage technology requires further analysis of their role in coping with climate change.

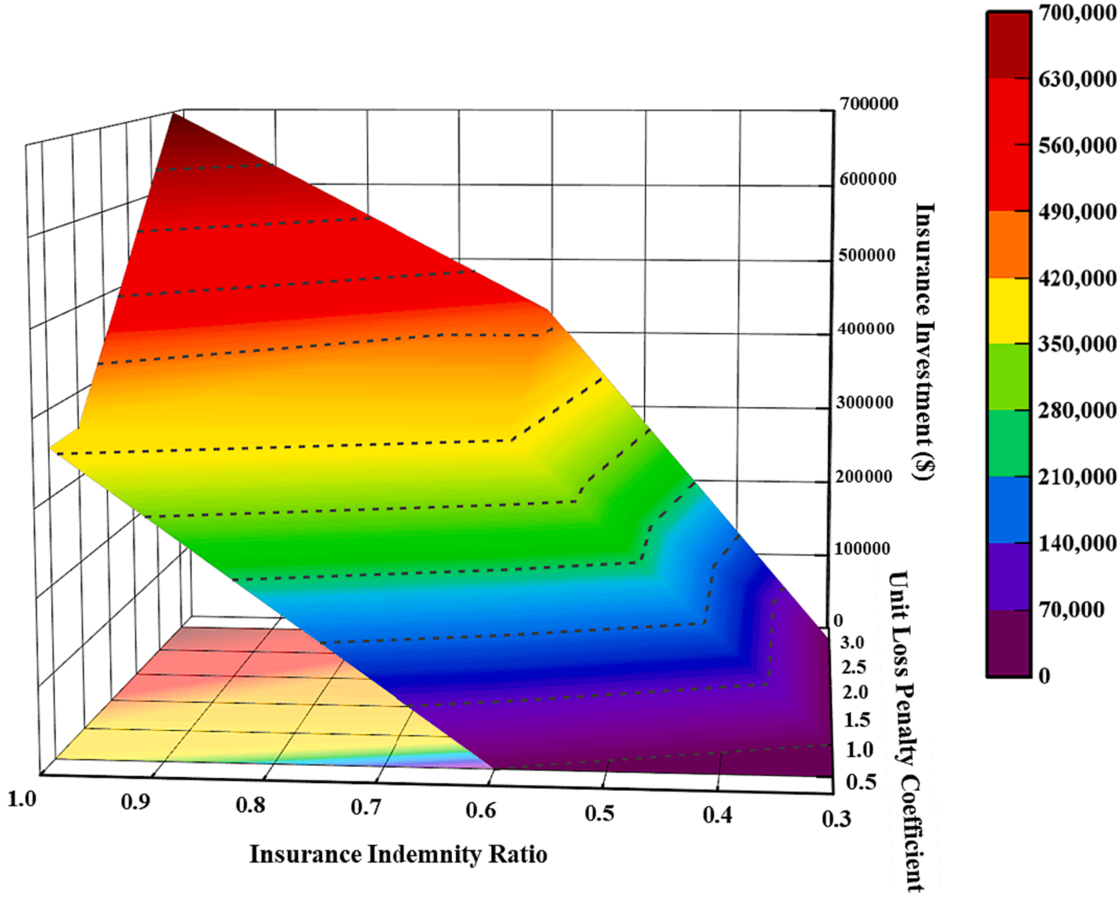

Fig. 9. Insurance investment under different degree of losses and proportion of insurance indemnity.

Table 3
ESS planning results under different cost coefficient.

α	ESS capacity (MWh)	ESS power (MW)
0.6	1.076	0.108
1.0	0.631	0.101
1.5	0.13	0.043

Table 4
Computational efficiency of two solving methods under different scenarios number (seconds).

Solution Methods	Scenarios Number		
	10	20	50
PH algorithm	2778	8625	17,963
Gurobi 9.5.2	1095	13459	N/A

CRediT authorship contribution statement

Qianwen Hu: Methodology, Software, Writing – original draft. **Gengfeng Li:** Conceptualization, Writing – review & editing. **Siyuan Sun:** Investigation, Data curation, Validation. **Zhaohong Bie:** Resources, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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